Sanjay Ghodawat University, Kolhapur

2020-21

Established as State Private University under Govt. of Maharashtra. Act No XL, 2017

EXM/P/09/00

Year and Program: S.Y B.Sc. **School of Science** **Department: Mathematics**

Course Code: MTS201 **Course Title: Mathematics III** **Semester : III**

Day : Friday **End Semester Examination** **Time:2 Hrs.**

Date: 18.12.2020 **Max Marks:50**

Instructions: 1) All Questions are compulsory.

		Marks	Bloom's Level	CO
Q.1	**Answer the following (any Two)–**			
a)	Prove that The set Q of all rational numbers is countable.	8	L_3	CO1
b)	Show that for each $n \in N$, the sum of the squares of the first n natural numbers is given by $\frac{1}{6}n(n+1)(2n+1)$	8	L_1	CO1
c)	If x and y are any real numbers with property $x < y$,then there exist a rational number $r \in \square$ such that $x < r < y$	8	L_3	CO2
Q.2	**Answer the following (any two) –**			
a)	State and prove Archimedean property.	8	L_3	CO2
b)	Show that, A sequence of real numbers is Cauchy if and only if it is convergent sequence.	8	L_4	CO3
c)	Let (x_n) be a sequence of positive real numbers such that $L = \lim_{n\to\infty}\left(\frac{x_{n+1}}{x_n}\right)$ exists. If $L < 1$,then $X = (x_n)$ converges and $\lim_{n\to\infty}(x_n) = 0$.	8	L_3	CO3
Q.3	**Answer the following (any two) –**			
a)	If a series in $\mathbf{R}$ is absolutely convergent, then prove that it is convergent.	9	L_2	CO4
b)	i) Let $\sum x_n$ be an absolutely convergent series in $\square$. Then show that any rearrangement $\sum y_k$ of $\sum x_n$ converges to same value.	4	L_1	CO4
	ii) A sequence (f_n) of bounded functions on $A \subseteq \square$ converges uniformly on A to f if $\|f_n - f\|_A \to 0$.	5	L_1	CO5

c) Let (f_n) be a sequence of functions in $\Re[a,b]$ and suppose that (f_n) converges uniformly on $[a,b]$ to f. Then show that $f \in \Re[a,b]$ and $\int_a^b f = \lim_{n\to\infty} \int_a^b f_n$. 9 L_1 CO5

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